Bandpass Filters Using Parallel Coupled Stripline Stepped Impedance Resonators

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Abstract—**Approximate design formulas for bandpass filters using parallel coupled stripline stepped impedance resonators (SIR) are derived. The formulas take into account the arbitrary coupling length as well as quarter-wavelength coupling. Some advantages of this filter are its abilities to control spurious response and insertion loss by changing the structure of the resonator. Using the design formulas two experimental filters were designed and fabricated and their performances closely matched design data.**

I. Introduction

The center frequency of the second passband of bandpass filters with half-wavelength resonators is two or three times that of the fundamental frequency. This results in poor harmonic suppression when used as output filters in oscillators or amplifiers. To overcome this problem, the use of nonuniform line resonators as a filter element was considered [1]. The tapered transmission line resonator is well known as one of these resonators, but it is not sufficiently practical for multistage bandpass filter application.

This paper describes a stepped impedance resonator (SIR) [2] used as a nonuniform transmission line resonator and its application to bandpass filters with the stripline construction.

Firstly, fundamental parameters such as resonance characteristics of an SIR and slope parameters are derived. Secondly, the design formulas of a bandpass filter was derived from the admittance inverter which is in a coupling circuit with arbitrary coupled electrical length. Finally, to verify the design formulas two experimental bandpass filters were designed and fabricated. The experimental performance data are shown to be in close agreement with the design data.

II. Resonance Properties of SIR

This section discusses the conditions of fundamental and spurious resonance of the SIR. The resonator structure to be considered here is shown in Fig. 1. The SIR is symmetrical and has two different characteristic impedance lines, Z_1 and Z_2, of admittance Y_1 and Y_2.

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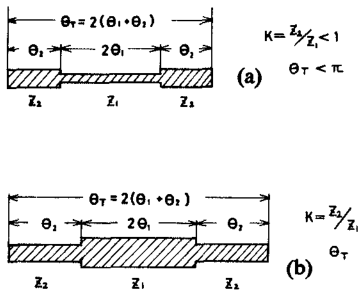

Fig. 1. Structures of the SIR.

A. Fundamental Resonance Condition

The admittance of the resonator from the open end, Y_i is given as

$$Y_i = jY_2 \cdot \frac{2(K\tan\theta_1 + \tan\theta_2)\cdot(K - \tan\theta_1\cdot\tan\theta_2)}{K(1-\tan^2\theta_1)\cdot(1-\tan^2\theta_2) - 2(1+K^2)\cdot\tan\theta_1\cdot\tan\theta_2} \tag{1}$$

where K = impedance ratio $= Z_2/Z_1$. The resonance condition can be obtained from the following:

$$Y_i = 0. \tag{2}$$

From (1) and (2) the fundamental resonance condition can be expressed as

$$K = \tan\theta_1 \cdot \tan\theta_2. \tag{3}$$

The relationship between θ_T and θ_1 is derived from (3) as

$$\tan\frac{\theta_T}{2} = \frac{1}{1-K}\cdot\left(\frac{K}{\tan\theta_1} + \tan\theta_1\right) \qquad (\text{when } K \neq 1) \tag{4}$$

$$\theta_T = \pi \qquad (\text{when } K = 1). \tag{5}$$

When $K = 1$, this corresponds to a uniform impedance line resonator.

It is evident from Fig. 2 that the resonator length θ_T has minimum value when $0 < K < 1$ and maximum value when $K > 1$. This condition can be obtained by differentiating (4) by θ_1,

$$\frac{1}{1-K}\cdot(\tan^2\theta_1 - K)\cdot\sin^2\theta_1 = 0 \tag{6}$$

then

$$\theta_1 = \tan^{-1}(\sqrt{K}) = \theta_2. \tag{7}$$

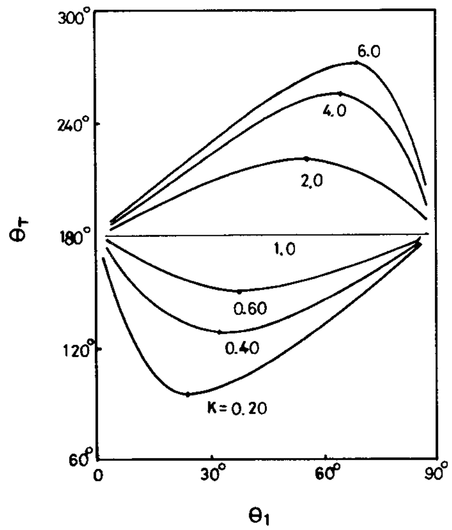

Fig. 2. Resonance condition of the SIR.

The above equation is the condition that θ_T has maximum or minimum value for constant K. For practical application it is preferable to choose $\theta_1 = \theta_2$ because the design equations can be simplified considerably. Therefore, in the following discussion, the SIR is treated as having $\theta_1 = \theta_2 = \theta$, and (1) can be expressed as

$$Y_i = jY_2 \frac{2(1+K)\cdot(K-\tan^2\theta)\cdot\tan\theta}{K-2(1+K+K^2)\cdot\tan^2\theta + K\tan^4\theta}. \tag{8}$$

The resonance condition is then given, using the fundamental frequency f_0 and corresponding length θ_0, as

$$\tan^2\theta_0 = K$$

or

$$\theta_0 = \tan^{-1}\sqrt{K}. \tag{9}$$

B. Spurious Resonance Frequency

Taking the spurious resonance frequency to be $f_{sn}(n = 1,2,3,\cdots)$ and corresponding θ with $\theta_{sn}(n=1,2,3,\cdots)$, we obtain from (8) and (2)

$$\tan\theta_{s1} = \infty$$
$$\tan^2\theta_{s2} - K = 0$$
$$\tan\theta_{s3} = 0 \tag{10}$$

then

$$\frac{f_{s1}}{f_0} = \frac{\theta_{s1}}{\theta_0} = \frac{\pi}{2\tan^{-1}\sqrt{K}}$$
$$\frac{f_{s2}}{f_0} = \frac{\theta_{s2}}{\theta_0} = 2\left(\frac{f_{s1}}{f_0}\right) - 1$$
$$\frac{f_{s3}}{f_0} = \frac{\theta_{s3}}{\theta_0} = 2\left(\frac{f_{s1}}{f_0}\right). \tag{11}$$

The above results are shown in Fig. 3 as a function of the impedance ratio K. It becomes evident from Fig. 3 that the spurious response can be controlled by the impedance ratio K, and this is one of the special features of the SIR.

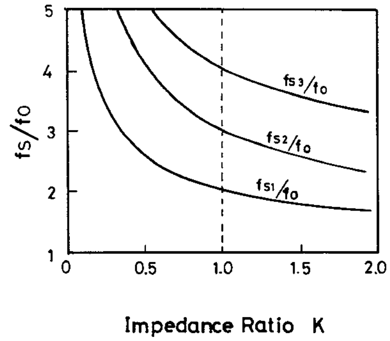

Fig. 3. Spurious resonance frequency of the SIR.

III. Susceptance Slope Parameter of the SIR

As a basis for establishing the resonance properties of resonators regardless of their form it is convenient to specify their resonant frequency ω_0 and their slope parameter [4]. For any resonator involving a shunt-type resonator the susceptance slope parameter can be expressed as

$$b = \frac{\omega_0}{2}\cdot\frac{dB}{d\omega}\bigg|_{\omega=\omega_0} \tag{12}$$

where B is the susceptance of the resonator. For the SIR considered here (12) becomes

$$b = \frac{\theta_0}{2}\cdot\frac{dB}{d\theta}\bigg|_{\theta=\theta_0}. \tag{13}$$

Differentiating (8) by θ, we obtain the following simple equation:

$$b = \frac{\theta_0}{2}\cdot 2(1+K)\cdot\frac{2}{1+K}\cdot Y_2$$
$$= 2\theta_0 Y_2. \tag{14}$$

The slope parameter for a half-wavelength resonator with uniform characteristic impedance $Z_0(=1/Y_0)$ can be induced from (14) in the case of $\theta_0 = \pi/4$ and $Y_2 = Y_0$, and we obtain the following well-known equation as [3], [4]

$$b = \frac{\pi}{2}Y_0. \tag{15}$$

IV. Parallel Coupled Section

This section considers a parallel coupled transmission line with arbitrary length and its equivalent circuit. For designing bandpass filters with SIR in which lines are coupled in parallel, it is necessary to find the relationship between even and odd mode impedance in the parallel coupled sections and the admittance inverter parameters [3].

A. Parallel Coupled Section as an Admittance Inverter

Fig. 4 (a) shows even and odd mode impedance Z_{oe}, Z_{oo} of a coupled line of electrical length θ, and the equivalent circuit is expressed by two single transmission lines of electrical length θ, impedance Z_0 and admittance

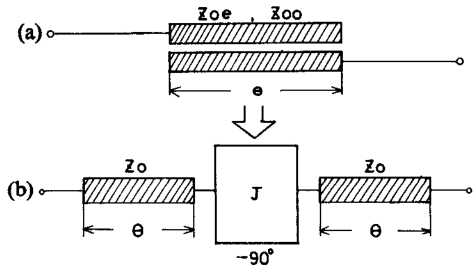

Fig. 4. Parallel coupled line and its equivalent with an inverter.

inverter parameter J, as shown in Fig. 4 (b). The $ABCD$ matrix for Fig. 4 (a) and (b) can be expressed as

$$[F_a]=\begin{bmatrix} \dfrac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta & j\dfrac{(Z_{oe}-Z_{oo})^2+(Z_{oe}+Z_{oo})^2\cdot\cos^2\theta}{2(Z_{oe}-Z_{oo})\cdot\sin\theta} \\ j\dfrac{2\sin\theta}{Z_{oe}-Z_{oo}} & \dfrac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta \end{bmatrix} \quad (16)$$

$$[F_b]=\begin{bmatrix} \left(JZ_0+\dfrac{1}{JZ_0}\right)\cdot\sin\theta\cdot\cos\theta & j\left(JZ_0^2\sin^2\theta-\dfrac{1}{J}\cos^2\theta\right) \\ j\left(\dfrac{1}{JZ_0^2}\sin^2\theta-J\cos^2\theta\right) & \left(JZ_0+\dfrac{1}{JZ_0}\right)\cdot\sin\theta\cdot\cos\theta \end{bmatrix}. \quad (17)$$

Then equalizing each corresponding matrix element, we can obtain

$$\frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta=\left(JZ_0+\frac{1}{JZ_0}\right)\cdot\sin\theta\cdot\cos\theta \quad (18)$$

$$\frac{(Z_{oe}-Z_{oo})^2+(Z_{oe}+Z_{oo})^2\cdot\cos^2\theta}{2(Z_{oe}-Z_{oo})\sin\theta}=JZ_0^2\sin^2\theta-\frac{1}{J}\cos^2\theta \quad (19)$$

$$\frac{2\sin\theta}{Z_{oe}-Z_{oo}}=\frac{1}{JZ_0^2}\cdot\sin^2\theta-J\cos^2\theta. \quad (20)$$

The above simultaneous equations are not independent of each other, and any two equations among the three are valid for solution. Solving (18) and (20), we obtain

$$\frac{Z_{oe}}{Z_0}=\frac{1+(J/Y_0)\operatorname{cosec}\theta+(J/Y_0)^2}{1-(J/Y_0)^2\cot^2\theta} \quad (21)$$

$$\frac{Z_{oo}}{Z_0}=\frac{1-(J/Y_0)\operatorname{cosec}\theta+(J/Y_0)^2}{1-(J/Y_0)^2\cot^2\theta}. \quad (22)$$

These are generalized expressions for parallel coupled lines with arbitrary length.

In the special case of quarter-wavelength coupling, by considering the situation $\theta=\pi/2$ in (21) and (22) then the following can be obtained:

$$\frac{Z_{oe}}{Z_0}=1+\left(\frac{Z_0}{K}\right)+\left(\frac{Z_0}{K}\right)^2 \quad (23)$$

$$\frac{Z_{oo}}{Z_0}=1-\left(\frac{Z_0}{K}\right)+\left(\frac{Z_0}{K}\right)^2 \quad (24)$$

where

$Z_0/K=J/Y_0$ (K= impedance inverter parameter).

The above two equations coincide with Cohn's results [3].

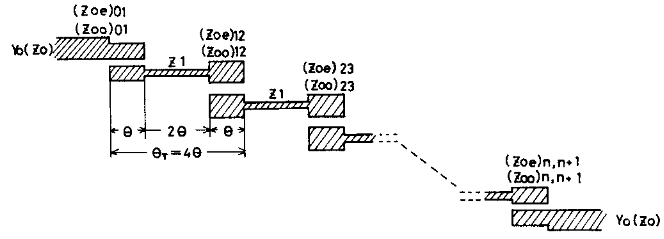

Fig. 5. Bandpass filter structure using the SIR.

B. Admittance Inverter Parameters for Bandpass Filters

The fundamental configuration of an n-stage bandpass filter considered here is shown in Fig. 5, and slope parameters for all resonators are of equal value, $b(b=2\theta_0 Y_0)$. When element values g_j and relative bandwidth w are given as fundamental design parameters of a bandpass filter [4], the admittance inverter parameter $J_{j,j+1}$ can be expressed as

$$J_{01}=\sqrt{\frac{Y_0 b_1 w}{g_0 g_1}}=Y_0\sqrt{\frac{2w\theta_0}{g_0 g_1}}$$

$$J_{j,j+1}=w\sqrt{\frac{b_j b_{j+1}}{g_j g_{j+1}}}=Y_0\frac{2w\theta_0}{\sqrt{g_j g_{j+1}}} \qquad (j=1\sim n-1)$$

$$J_{n,n+1}=\sqrt{\frac{Y_0 b_1 w}{g_n g_{n+1}}}=Y_0\sqrt{\frac{2w\theta_0}{g_n g_{n+1}}}. \quad (25)$$

Using (21) and (22) in the previous section, the design data for coupling lines can be obtained. It is then possible to design a bandpass filter as a SIR.

V. Experimental Filters

This section describes the design and performance of experimental filters. Two kinds of filters were designed and fabricated to verify the design formulas discussed above. The impedance ratio K was chosen to be 0.5 for the Type 1 filter and 1.5 for the Type 2 filter.

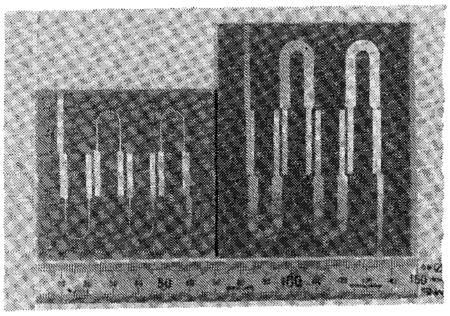

Fig. 6. Photograph of the experimental SIR bandpass filters (left: Type 1, right: Type 2).

TABLE I
VARIOUS PARAMETERS FOR DESIGNING EXPERIMENTAL FILTERS

	Type - 1	Type - 2
Impedance Ratio K	0.50	1.50
Line Impedance		
Z_1 (Ω)	100	33.3
Z_2 (Ω)	50	50
Resonator Length θ_T (deg.)	141	203
Slope Parameter b (Mho)	0.0246	0.0354
Even, Odd Mode Impedance (Ω)		
$(Zoe)_{01}$ = $(Zoe)_{45}$	88.43	80.67
$(Zoo)_{01}$ = $(Zoo)_{45}$	35.61	37.08
$(Zoe)_{12}$ = $(Zoe)_{34}$	55.06	55.46
$(Zoo)_{12}$ = $(Zoo)_{34}$	45.79	45.53
$(Zoe)_{23}$	53.63	53.91
$(Zoo)_{23}$	46.83	46.62

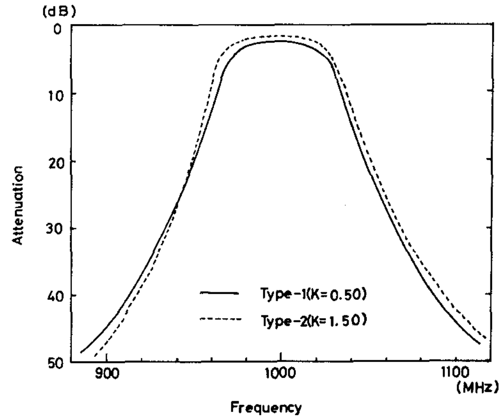

Fig. 7. Measured frequency response of the experimental filters.

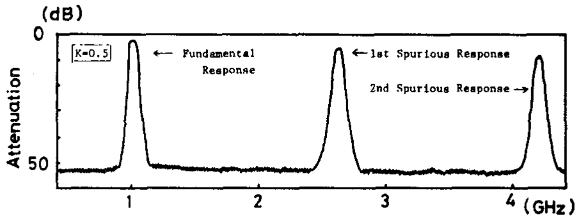

Fig. 8. Measured spurious response of the Type 1 filter.

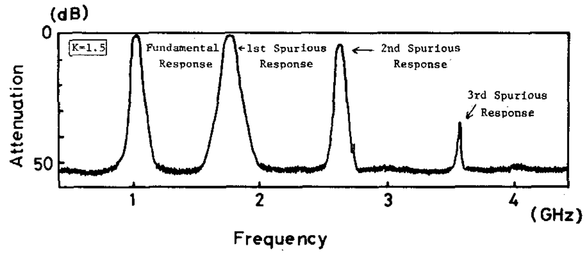

Fig. 9. Measured spurious response of the Type 2 filter.

A. Design Parameters

On the basis of the derived formulae two experimental filters were designed using the following parameters:

center frequency	$f_0 = 1.00$ GHz
number of resonators	$N = 4$
response	Chebyshev
passband ripple	$R = 0.01$ dB
relative bandwidth	$w = 0.04$

and impedance ratio K

$K = 0.50$ (for Type 1 filter)

$K = 1.50$ (for Type 2 filter).

Various calculated design parameters are listed in Table I.

The filters were fabricated with a substrate having a dielectric constant of $\epsilon_r = 2.6$, and a triplate strip-line structure [5] in which the distance between ground planes was 3.15 mm. The arrangement of filters is shown in Fig. 6. Each resonator has a hairpin structure [6], but spacing between line pairs constituting the hairpin resonator is wide enough to discount coupling between the line pairs. The actual dimensions of the resonator must be determined by taking into consideration end and junction effects of striplines [7], [8].

B. Performance of Experimental Filters

Fig. 7 shows the attenuation characteristics of experimental filters. The measured fractional bandwidths are slightly more (around 10 percent) than the designed values. The midband insertion losses are 2.8 dB and 2.0 dB for Type 1 ($k = 0.5$) and Type 2 ($k = 1.5$), respectively. This is caused by a difference in the stripline width of the Z_1 section where current has maximum value.

The spurious responses of experimental filters are shown in Fig. 8 and Fig. 9 for $K = 0.5$ and 1.5, respectively. There is no distinct response around $2f_0$, $3f_0$, and $4f_0$ in both figures, and these filters are considered to be effective for harmonic suppression as output filters of amplifiers or oscillators. The measured center frequency of the spurious passbands are listed in Table II compared with calculated

TABLE II
CALCULATED AND MEASURED VALUES OF SPURIOUS RESONANCE FREQUENCY

Center frequency of the spurious passband	Type-1 (K=0.50)		Type-2 (K=1 50)	
	Calculated	Measured	Calculated	Measured
f_{s1}	2.55 fo	2 60 fo	1.77 fo	1 75 fo
f_{s2}	4 10 fo	4.20 fo	2 55 fo	2 55 fo
f_{s3}	511 fo	5 20 fo	3 55 fo	3.55 fo

values, and it shows that there is close agreement between them.

From the above experimental results it must be noted that the performance of bandpass filters using SIR depends on its structures, or the K value. In design practice it is desirable to choose a value for K of less than 1 if compactness or wide stop band characteristics are desired, and it is appropriate to choose a K value of more than 1 if low loss characteristics are important.

VI. Conclusions

A method of designing bandpass filters suitable for striplines with stepped impedance resonators (SIR) was established and fabricated filter performance closely coincided with design data.

A special feature of this filter is that the spurious response and insertion loss can be controlled by the impedance ratio K of the resonator.

Design equations for a conventional filter [3] using a parallel coupled stripline resonator, which has uniform line and quarter wavelength coupling, can be obtained from equations presented here as a special case in which $K=1.0$ and coupling length $\theta=90°$. Therefore, the design method described in this paper is considered to involve generalized equations for parallel coupled stripline resonator filters.

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